Spin and density longitudinal response of quantum dots in time-dependent local-spin-density approximation

Ll. Serra[1], M. Barranco[2], A. Emperador[2], M. Pi[2], and E. Lipparini[3].

[1]*Departament de Física, Facultat de Ciències,*
Universitat de les Illes Balears, E-07071 Palma de Mallorca, Spain
[2]*Departament d'Estructura i Constituents de la Matèria, Facultat de Física,*
Universitat de Barcelona, E-08028 Barcelona, Spain
[3]*Dipartimento di Fisica, Università di Trento, and INFM sezione di Trento, 38050 Povo, Italy*
(February 1, 2008)

Abstract

The longitudinal dipole response of a quantum dot has been calculated in the far-infrared regime using local spin density functional theory. We have studied the coupling between the collective spin and density modes as a function of the magnetic field. We have found that the spin dipole mode and single particle excitations have a sizeable overlap, and that the magnetoplasmon modes can be excited by the dipole spin operator if the dot is spin polarized. The frequency of the dipole spin edge mode presents an oscillation which is clearly filling factor (ν) related. We have found that the spin dipole mode is especially soft for even ν values, becoming unstable for magnetic fields in the region $1 < \nu \leq 2$. Results for selected number of electrons and confining potentials are discussed. An analytical model which reproduces the main features of the microscopic spectra has been developed.

PACS 73.20.Dx, 72.15.Rn

Typeset using REVTEX

I. INTRODUCTION

The far-infrared (FIR) response of quantum dots is a subject of current interest since the experiments carried out by Sikorski and Merkt [1], and by Demel et al [2]. These experiments and the subsequent theoretical work (see Refs. [3–8] and references therein) have shown that the excitation spectrum of quantum dots in the FIR region is dominated by the dipole edge magnetoplasmon peak that splits into two different B dispersion branches when a magnetic field B is applied perpendicularly to the dot. These peaks are density (charge) collective modes excited by the operator $D_\rho = \sum_{i=1}^{N} x_i$. In the case of harmonic confinement by the potential $\frac{1}{2} m \omega_0^2 r^2$, as a consequence of Kohn's theorem [9] the density mode is not coupled to any other mode, and the dipole operator D_ρ excites only two collective states at the energies $\omega_\pm = \sqrt{\omega_0^2 + \frac{1}{4} \omega_c^2} \pm \frac{1}{2} \omega_c$, where ω_c is the cyclotron frequency. If the confining potential is not harmonic, Kohn's theorem does not hold, and on the one hand the energy of the modes depend on the number of electrons in the dot, and on the other hand a richer excitation spectrum appears.

Raman spectroscopy has made possible to observe in the same sample single particle (sp), charge and spin density excitations [10,11], whose evolution as a function of B has been studied in recent experiments. This has revealed several interesting features of the sp [12] and of the spin collective excitations [13] in quantum dots. Limiting to the later, the experiments have determined that the spin mode lies very close in energy to the uncorrelated single electron excitations, and that magnetoplasmons can also be detected using spin-dependent probes. Besides, it has been experimentally determined that the spin mode has a much lower energy than the charge mode. These facts constitute the body of experimental results that any theory aiming at a quantitative simultaneous description of spin and charge density collective modes in quantum dots should reproduce.

The dipole spin response function for unpolarized quantum dots at zero magnetic field has been recently addressed by two of us [14]. In the FIR regime, it has been found that the response is dominated by a low-energy collective dipole spin mode excited by the operator $D_m = \sum_{i=1}^{N} x_i \sigma_z^i$, where x_i and σ_z^i are Cartesian components of the position and spin vectors, and N is the number of electrons in the dot. Similar modes have been described in atomic nuclei [15], and in alkali metal clusters [16,17].

The aim of the present work is to extend our previous study to the case of a quantum dot submitted to a perpendicular static magnetic field, which originates a B dependent spin polarization in the ground state of the dot. We will explicitly show that this not only causes the splitting of the spin dipole mode into two branches, one with negative and another with positive B dispersion, but also its coupling with the dipole density mode mainly excited by the operator D_ρ. We shall see that if the confinement is not harmonic and the dot is polarized, that operator also excites the dipole spin mode. Viceversa, when the dot is polarized, which is the case if it has an odd number of electrons, or for most cases when B acts on the dot, the spin response is coupled to the density response so that the external operator $D_m = \sum_i x_i \sigma_z^i$ also excites the density mode. When the system is fully polarized both modes coincide, while at zero polarization they are uncoupled.

To this end, we have self-consistently evaluated the longitudinal response of the dot in

the time-dependent local-spin-density approximation (TDLSDA). As longitudinal we mean an external field which is either spin independent, or dependent on the spin component parallel to the magnetic field, i.e., the z component. We present results obtained for selected numbers of electrons and confining potentials. Specifically, we have used a harmonic oscillator potential to describe dots with $N = 5$ and 25, and a disk confining potential to describe dots with 25 and 210 electrons, for which the FIR response has been determined in detail [2]. The results obtained for the $N = 5$ dot have been presented as preliminary results in Ref. [18]. The ground state (gs) structure of the later two dots in intense magnetic fields has been recently addressed [19,20]. However, no self-consistent TDLSDA calculation for a dot as large as $N = 210$ has been carried out before even in the density channel.

To obtain correct collective modes one needs to have a proper description of the ground state these excitations are built on. Several density functional calculations have addressed this question [21–25]. The LSDA we have used in the present work is based on the exchange-correlation energy functional employed in Ref. [21] as an input to construct their current density functional theory (CDFT). Within the range of magnetic fields we are interested in, we have checked that both LSDA and CDFT are yielding similar results for gs properties other than the current density. Tests of CDFT against exact and Hartree-Fock (HF) calculations have been presented in Ref. [21]. Tests of unrestricted HF against exact gs energies in the filling factor region $2 \geq \nu \geq 1$ have also been presented in Ref. [26] for small number of electrons (up to 5).

We conclude from the comparisons presented in the above references, that TDLSA may yield fairly accurate results for the density and spin response in the range of magnetic fields for which experimental information is currently available. Comparison with these experiments constitute the ultimate test of this essentially parameter-free approximation. Between $1/2 \leq \nu \leq 1$, the results of Refs. [23,25] seem to indicate that the exchange-correlation energy of Ref. [21] is not very suitable because it is based on the Levesque-Weiss-MacDonald interpolation formula [27], and improved exchange-correlation energy functionals like those of Ref. [23,25] are definitely better off (a detailed account of the accuracy of elaborated LSDA's in the $\nu \leq 1$ regime is also given in Ref. [25]). It remains to be seen whether these improvements are crucial to describe the experimental data which for the moment are limited to comparatively small magnetic fields.

To get a deeper understanding, simple methods have been developed to reproduce the gross features of the microscopic spectra. One such method is the sum rule approach used in Ref. [19] to describe multipole density modes in quantum dots. We present here an analytical model, called vibrating potential model (VPM), which allows to understand the TDLSDA results in a clear way, thereby providing physical insight onto the longitudinal response of quantum dots. The VPM model has been widely used in nuclear physics [28], where it was developed to describe nuclear collective modes. It has also been applied to the description of simple metal clusters [29].

II. THE LONGITUDINAL RESPONSE

We consider a quantum dot made of N electrons moving in the $z = 0$ plane where they are confined by the circular potential $V^+(r)$ in the presence of a constant magnetic field B

in the z direction. In the local-spin-density approximation (LSDA), the single electron wave functions are given by the solution of the Kohn-Sham (KS) equations

$$\left[-\frac{1}{2}\nabla^2 + \frac{1}{2}\omega_c \ell_z + \frac{1}{8}\omega_c^2 r^2 + V^+(r) \right.$$
$$\left. + V^H + V^{xc} + (W^{xc} + \frac{1}{2}g^*\mu_B B)\sigma_z \right] \varphi_\alpha(r,\theta) = \epsilon_\alpha \varphi_\alpha(r,\theta) \ , \tag{1}$$

where $V^H = \int d\vec{r}' \rho(\vec{r}')/|\vec{r}-\vec{r}'|$ is the Hartree potential. $V^{xc} = \partial\mathcal{E}_{xc}(\rho,m)/\partial\rho|_{gs}$ and $W^{xc} = \partial\mathcal{E}_{xc}(\rho,m)/\partial m|_{gs}$ are the variations of the exchange-correlation energy density $\mathcal{E}_{xc}(\rho,m)$ in the local approximation taken at the ground state, and $\rho(r)$ and $m(r)$ are the electron and spin magnetization densities. The exchange-correlation energy density $\mathcal{E}_{xc}$ has been constructed from the results of Ref. [30] on the nonpolarized and fully polarized two dimensional (2D) electron gas in the same way as in Ref. [31], i.e., using the von Barth and Hedin [32] prescription to interpolate between both regimes.

We have used effective atomic units ($\hbar = e^2/\epsilon = m = 1$), where ϵ is the dielectric constant of the semiconductor and m is the electron effective mass. In units of the bare electron mass m_e one has $m = m^* m_e$. In this system of units, the length unit is the effective Bohr radius $a_0^* = a_0 \epsilon/m^*$, and the energy unit is the effective Hartree $H^* = H m^*/\epsilon^2$. For GaAs we have taken $\epsilon = 12.4$, $m^* = 0.067$, and $g^* = -0.44$, which yields $a_0^* = 97.9$ Å and $H^* \sim 11.9$ meV ~ 95.6 cm^{-1}. In Eq. (1) $\omega_c = eB/(mc)$ is the cyclotron frequency and $\mu_B = e\hbar/(2m_e c)$ is the Bohr magneton. The use of the same letter for the effective mass and the spin magnetization, and for the dielectric constant and the single electron energies should cause no confusion, since neither the mass nor the dielectric constant will explictly appear in the rest of the work.

As a consequence of circular symmetry the φ_α's are eigenstates of the orbital angular momentum ℓ_z, i.e., $\varphi_\alpha(r,\theta) = u_{n\ell\sigma}(r)e^{-i\ell\theta}$, with $\ell = 0, \pm 1, \pm 2, \ldots$. The gs electron density is given by $\rho(r) = \sum_\alpha n_\alpha |u_\alpha(r)|^2$, while the gs spin magnetization density is expressed in terms of the spin of orbital α, $\langle\sigma_z\rangle_\alpha$, as $m(r) = \sum_\alpha n_\alpha \langle\sigma_z\rangle_\alpha |u_\alpha(r)|^2$. The numerical calculations reported in the following have been performed at a small but finite temperature $T \leq 0.1$ K, and the KS equations have been solved by integration in r space. The thermal occupation probabilities n_α are determined by the normalization condition

$$N = \sum_\alpha n_\alpha = \sum_\alpha \frac{1}{1 + \exp[(\epsilon_\alpha - \mu)/k_B T]} \tag{2}$$

which fixes the chemical potential μ. Our iterative method works for weak and strong magnetic fields as well, for which the effective potential is very different. It has probed to be very robust, allowing us to handle without any problem several hundreds of electrons.

Once the gs has been obtained, we determine the induced densities originated by an external field employing linear-response theory. Following Refs. [33,34], we can write the variation $\delta\rho_\sigma$ induced in the spin density ρ_σ ($\sigma \equiv \uparrow, \downarrow$) by an external spin-dependent field F, whose non-temporal dependence we denote as $F = \sum_\sigma f_\sigma(\vec{r}) |\sigma\rangle\langle\sigma|$:

$$\delta\rho_\sigma(\vec{r},\omega) = \sum_{\sigma'} \int d\vec{r}' \chi_{\sigma\sigma'}(\vec{r},\vec{r}';\omega) f_{\sigma'}(\vec{r}') \ , \tag{3}$$

where $\chi_{\sigma\sigma'}$ is the spin-density correlation function. In this limit, the frequency ω corresponds to the harmonic time dependence of the external field F and of the induced $\delta\rho_\sigma$. Eq. (3) is

a 2×2 matrix equation in the two-component Pauli space. In longitudinal response theory, F is diagonal in this space, and we write its diagonal components as a vector $F \equiv \begin{pmatrix} f_\uparrow \\ f_\downarrow \end{pmatrix}$. For the operators defined at the Introduction we then have

$$D_\rho \equiv \begin{pmatrix} x \\ x \end{pmatrix} \quad \text{and} \quad D_m \equiv \begin{pmatrix} x \\ -x \end{pmatrix} \quad . \tag{4}$$

The TDLSDA assumes that electrons respond as free particles to the perturbing effective field, which consists of the external plus the induced field arising from the changes produced by the perturbation in the gs mean field. This condition defines the TDLSDA correlation function $\chi_{\sigma\sigma'}$ in terms of the free-particle spin-density correlation function $\chi_{\sigma\sigma'}^{(0)}$ through a Dyson-type integral equation:

$$\chi_{\sigma\sigma'}(\vec{r}, \vec{r}'; \omega) = \chi_{\sigma\sigma'}^{(0)}(\vec{r}, \vec{r}'; \omega)$$
$$+ \sum_{\sigma_1\sigma_2} \int d\vec{r}_1 d\vec{r}_2 \, \chi_{\sigma\sigma_1}^{(0)}(\vec{r}, \vec{r}_1; \omega) K_{\sigma_1\sigma_2}(\vec{r}_1, \vec{r}_2) \chi_{\sigma_2\sigma'}(\vec{r}_2, \vec{r}'; \omega) \quad . \tag{5}$$

The free particle spin-correlation function at finite temperature is obtained from the KS sp wave functions, energies and occupation probabilities:

$$\chi_{\sigma\sigma'}^{(0)}(\vec{r}, \vec{r}', \omega) = -\delta_{\sigma,\sigma'} \sum_{\alpha\beta} \varphi_\alpha^*(\vec{r}) \varphi_\beta(\vec{r}) \frac{n_\alpha - n_\beta}{\epsilon_\alpha - \epsilon_\beta + \omega + i\eta} \varphi_\beta^*(\vec{r}') \varphi_\alpha(\vec{r}') \quad . \tag{6}$$

The label α (β) refers to a sp level with spin σ (σ') and occupation probability n_α (n_β). To simplify the analysis of the results, we have added a small but finite imaginary part η to the energy ω. This will make an average of the strength function by transforming the δ-peaks into Lorentzians of width 2η.

The kernel $K_{\sigma\sigma'}(\vec{r}, \vec{r}')$ is the residual two-body interaction

$$K_{\sigma\sigma'}(\vec{r}_1, \vec{r}_2) = \frac{1}{|\vec{r}_1 - \vec{r}_2|} + \left.\frac{\partial^2 \mathcal{E}_{xc}(\rho, m)}{\partial\rho_\sigma \partial\rho_{\sigma'}}\right|_{gs} \delta(\vec{r}_1 - \vec{r}_2) \,, \tag{7}$$

where

$$\left.\frac{\partial^2 \mathcal{E}_{xc}}{\partial\rho_\sigma \partial\rho_{\sigma'}}\right|_{gs} = \left.\frac{\partial^2 \mathcal{E}_{xc}}{\partial\rho^2}\right|_{gs} + (\eta_\sigma + \eta_{\sigma'}) \left.\frac{\partial^2 \mathcal{E}_{xc}}{\partial\rho \, \partial m}\right|_{gs} + \eta_\sigma \eta_{\sigma'} \left.\frac{\partial^2 \mathcal{E}_{xc}}{\partial m^2}\right|_{gs}$$
$$\equiv K(r) + (\eta_\sigma + \eta_{\sigma'}) \, L(r) + \eta_\sigma \eta_{\sigma'} \, I(r) \,, \tag{8}$$

with $\eta_\uparrow = 1, \eta_\downarrow = -1$. The last expression is the definition of the K, L, and I functions.

When the system is not polarized, there are only two independent correlation functions. These are $\chi_{\rho\rho}$ and χ_{mm} describing, respectively, the density response to D_ρ and the spin response to D_m. They are given by

$$\chi_{\rho\rho} = \chi_{\uparrow\uparrow} + \chi_{\downarrow\downarrow} + \chi_{\uparrow\downarrow} + \chi_{\downarrow\uparrow}$$
$$\chi_{mm} = \chi_{\uparrow\uparrow} + \chi_{\downarrow\downarrow} - \chi_{\uparrow\downarrow} - \chi_{\downarrow\uparrow} \,, \tag{9}$$

and the four equations (5) reduce to two uncoupled equations for $\chi_{\rho\rho}$ and χ_{mm} whose kernels are given by $1/r_{12} + K\delta(r_{12})$ and $I\delta(r_{12})$, respectively, and the free-particle correlation

function $\chi^{(0)} = \chi^{(0)}_{\uparrow\uparrow} + \chi^{(0)}_{\downarrow\downarrow} = 2\chi^{(0)}_{\uparrow\uparrow}$ is the same in both channels because $\chi^{(0)}_{\uparrow\uparrow} = \chi^{(0)}_{\downarrow\downarrow}$. This constitutes the paramagnetic limit of the longitudinal response with uncoupled density and spin channels [14], in which the residual interaction consists of a Coulomb direct plus an exchange-correlation terms in one case, and only of an exchange-correlation term in the other.

When the system is polarized one no longer has $\chi^{(0)}_{\uparrow\uparrow} = \chi^{(0)}_{\downarrow\downarrow}$, and there are two more independent correlation functions

$$\chi_{\rho m} = \chi_{\uparrow\uparrow} - \chi_{\downarrow\downarrow} - \chi_{\uparrow\downarrow} + \chi_{\downarrow\uparrow}$$
$$\chi_{m\rho} = \chi_{\uparrow\uparrow} - \chi_{\downarrow\downarrow} + \chi_{\uparrow\downarrow} - \chi_{\downarrow\uparrow} , \tag{10}$$

which produce the density response to D_m and the spin response to D_ρ, respectively.

Equations (5) have been solved as a generalized matrix equation in coordinate space after performing an angular decomposition of $\chi_{\sigma\sigma'}$ and $K_{\sigma\sigma'}$ of the kind

$$K_{\sigma\sigma'}(\vec{r}, \vec{r}') = \sum_\ell K^{(\ell)}_{\sigma\sigma'}(r, r') e^{i\ell(\theta - \theta')} . \tag{11}$$

Only modes with $\ell = \pm 1$ couple to the external dipole fields D_ρ and D_m. This can be readily seen performing the angular integral in Eq. (3). In practice, we have considered the multipole expansion of the external field, using the dipole vectors

$$D^{(\pm 1)}_\rho = \frac{1}{2} r e^{\pm i\theta} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } D^{(\pm 1)}_m = \frac{1}{2} r e^{\pm i\theta} \begin{pmatrix} 1 \\ -1 \end{pmatrix} . \tag{12}$$

For a polarized system having a non zero magnetization in the gs, the $\ell = \pm 1$ modes are not degenerate and give rise to two excitation branches with $\Delta L_z = \pm 1$, where L_z is the gs orbital angular momentum.

The response functions corresponding to the above dipole fields have been obtained from the $\ell = \pm 1$ components of the correlation functions $\chi^{(\pm 1)}_{AB}(r, r'; \omega)$ with $A, B = \rho, m$ as:

$$\alpha_{AB}(\omega) = \pi^2 \int dr_1 \, dr_2 \, r_1^2 \, r_2^2 \, (\chi^{(+1)}_{AB}(r_1, r_2; \omega) + \chi^{(-1)}_{AB}(r_1, r_2; \omega))$$
$$\equiv \alpha^{(+1)}_{AB}(\omega) + \alpha^{(-1)}_{AB}(\omega) \tag{13}$$

Their imaginary parts are related to the strength functions as $S_{AB}(\omega) = \frac{1}{\pi} \text{Im}[\alpha_{AB}(\omega)]$. Although the excitation energy ω and strength functions are always positive, it may be easily verifed that the following relations formally hold:

$$\text{Re}\left[\alpha^{(-\ell)}_{AB}(\omega)\right] = \text{Re}\left[\alpha^{(\ell)}_{AB}(-\omega)\right]$$
$$\text{Im}\left[\alpha^{(-\ell)}_{AB}(\omega)\right] = -\text{Im}\left[\alpha^{(\ell)}_{AB}(-\omega)\right] . \tag{14}$$

To check the numerical accuracy of the calculations we have used the f-sum rules for the dipole operators, which can be expressed in terms of gs quantities [35]:

$$m_1^{(\rho\rho)} = \int S_{\rho\rho}(\omega) \, \omega \, d\omega = \frac{1}{2}\langle 0|[D_\rho, [H, D_\rho]]|0\rangle = \frac{N}{2}$$
$$m_1^{(mm)} = \int S_{mm}(\omega) \, \omega \, d\omega = \frac{1}{2}\langle 0|[D_m, [H, D_m]]|0\rangle = \frac{N}{2}$$
$$m_1^{(m\rho)} = m_1^{(\rho m)} = \int S_{m\rho}(\omega) \, \omega \, d\omega + \int S_{\rho m}(\omega) \, \omega \, d\omega = \langle 0|[D_m, [H, D_\rho]]|0\rangle = 2S_z , \tag{15}$$

where S_z is the total spin of the ground state.

III. RESULTS

Figures 1-3 display the dipole strength function of the $N = 5$, 25, and 210 dot for selected B values. Solid lines correspond to the density response to D_ρ, and dotted lines to the spin response to D_m, that is to $S_{\rho\rho}$ and to S_{mm}. Dashed lines represent the free particle strength function. For the five electron dot we have used a parabolic confining potential $V^+(r) = \frac{1}{2}m\omega_0^2 r^2$ with $\omega_0 = 4.28$ meV, and for the other dots we have used a disk confining potential as described in Refs. [19,20].

For $N = 25$ and 210, most B values displayed correspond to integer filling factors ν. It was found in Ref. [20] that for $N = 210$ and an R disk confining potential, these values follow the law $\nu = 2\pi c\, n_s/(eB)$ pertaining to the 2D system, where $n_s = N/(\pi R^2)$ is the electron surface density. For the $N = 25$ dot, that law yields $B = 3.29$ T as the value at which the system becomes fully polarized. Actually, we have found that the dot is in the maximum density drop (MDD) state for 3.42 T $\leq B \leq$ 3.70 T. We have taken as $\nu = 1$ configuration that corresponding to $B_1 = 3.56$ T, and have defined the other ν configurations as those corresponding to the value $B_\nu = B_1/\nu$.

The results of Ref. [2] for the $N = 25$ dot seem to indicate that a confining potential of parabolic type might be more adequate (see the discussion of the charge mode at the end of this Section). Consequently, we have also studied it using a harmonic confining potential with $\omega_0 = 2.78$ meV to reproduce the experimental dipole energy at $B = 0$. In this case the system is in the MDD state for 4.46 T $\leq B \leq$ 4.69 T. We have taken $B_1 = 4.58$ T and have defined the other ν configurations as indicated before.

The existence of a MDD state for this dot is in agreement with the findings of Ref. [24]. It is also worth to remark that even for such a small dot, ν as defined before also coincides with the number of occupied $(n, \uparrow$ or $\downarrow)$ bands for values up to $\nu = 5$-6, which correspond to rather low B intensities. We present in Fig. 4 the strength function corresponding to $N = 25$ with parabolic confinement.

Figures 1-4 show that in both channels the response at $B = 0$ is concentrated within a small energy range, with one single peak or with several closely lying fragmented peaks which exhaust most of the f-sum rule. The peak energy is lower in the spin than in the density channel. This is due to the character of the residual interaction, which is attractive in the former and repulsive in the later channel, and shifts the TDLSDA responses from the free particle response in opposite directions. The residual interaction in the spin channel is weaker than in the density channel, where not only the exchange-correlation but also the Coulomb direct term contributes. Consequently, the spin response is close in energy to the free response. It is thus difficult to distinguish the collective spin mode from the single particle spectrum. In large dots it also causes a stronger Landau damping in the spin than in the density channel. These facts have been observed and discussed in Ref. [13].

At $B = 0$, as a consequence of Kohn's theorem, if the confining potential is harmonic with frequency ω_0 the excitation energy of the dipole density mode is equal to ω_0 irrespective of the number of electrons. Otherwise, the excitation energy depends on N, see for example Refs. [5,14]. In the spin channel Kohn's theorem does not hold, and a size dependence appears in the dipole spin mode even for parabolic confining potentials [14].

When B is not zero, the dipole mode in either channel splits into two branches, one with negative and another with positive B dispersion. The splitting is due to the breaking of the

ℓ-degeneracy of the sp energies by the applied magnetic field. Several new phenomena then appear. We first notice that for B values such that the spin of the dot gs is different from zero, the spin and density modes are coupled. This is particularly apparent in the $N = 210$ dot. Indeed, at $B = 1.71$ and 5.14 T the system is almost paramagnetic, having $2S_z = 2$ and 0, respectively (see Fig. 4 of Ref. [20]). As a consequence, the modes are uncoupled, as it can be seen from panel (b) and especially from panel (d) of Fig. 3. In contradistinction, at $B = 3.43$ and 7 T we have $2S_z = 54$ and 74: the system has a large spin magnetization in the gs and the spin and density modes are clearly coupled, as displayed in panels (c) and (e) of that figure. One can see a distinct peak in the spin response at the energy of the density mode. This effect has been experimentaly observed [13]. The strength of this peak increases with S_z and when the system is fully polarized, which happens sligthly above $B = 10$ T for the $N = 210$ dot, all the strength is transferred from the spin to the density channel. Conversely, the spin mode may be observed in the density channel with some intensity. This effect is hindered because Kohn's theorem prevents it from occuring for parabolic potentials, and for the disk potential it is of order $(2S_z/N)^2$.

The interplay between charge and spin modes is especially marked in the mixed channel where the density response to the spin dipole operator D_m, or the spin response to the density dipole operator D_ρ are described. This is shown in Fig. 5 for the N=210 dot at $\nu = 3$. One clearly observes two peaks at the energy of the density modes, and another two at the energy of the spin modes. It can be understood casting the mixed response into a sum over 'spin dipole states' $|m\rangle$ and another over 'charge dipole states' $|\rho\rangle$

$$
\begin{aligned}
S_{m\rho}(\omega) = S_{\rho m}(\omega) &= \sum_n \langle 0|D_\rho|n\rangle\langle n|D_m|0\rangle\delta(\omega - \omega_{n0}) \\
&= \sum_\rho \langle 0|D_\rho|\rho\rangle\langle\rho|D_m|0\rangle\delta(\omega - \omega_{\rho 0}) \\
&+ \sum_m \langle 0|D_\rho|m\rangle\langle m|D_m|0\rangle\delta(\omega - \omega_{m0}) \ .
\end{aligned}
\tag{16}
$$

For a disk confining potential, the matrix element $\langle 0|D_\rho|m\rangle$ is not zero and there is a contribution to $S_{m\rho}$ from the spin modes. For a harmonic confining potential $\langle 0|D_\rho|m\rangle$ is zero and only the density modes would contribute to $S_{m\rho}$ through the ρ-sum in Eq. (16).

The B dispersion of the main peaks of the spectrum is reported in Figs. 6−9. In these figures the density modes are represented by dots and the spin modes by triangles. Solid symbols correspond to integer filling factor values, and the inserts show the negative B dispersion branch of the spin mode. As a guide, we have drawn lines starting at the value of the $B = 0$ frequency and following the VPM B dispersion laws (see Eqs. 24).

Several features of these figures are worth to discuss. Concerning the spin modes, we first see that at low B their energy is much smaller than the energy of the density modes, in agreement with the experimental findings of Ref. [13]. At higher B the dot is eventually fully polarized and the longitudinal spin and density modes merge, as in the two dimensional electron gas (see Fig. 5 of Ref. [36]). This is not explicitly shown in the figures. Secondly, the negative B dispersion branch of the spin mode manifests a clear oscillatory behaviour with ν, similar to that found for the density response [37], also discussed in Ref. [20]: the 'paramagnetic' even ν configurations have softer spin modes than the 'ferromagnetic' odd ν configurations.

Our calculation predicts a spin instability occurring when the energy of the spin mode lower branch goes to zero at a critical B between $\nu = 1$ and 2. This instability also manifests in the static spin polarizability $\text{Re}[\alpha_{mm}^{(-1)}(0)]$, which becomes negative at these large B's. This indicates that the gs is no longer an energy minumum and thus is not stable. It is worth to recall that no collective spin dipole modes are observed in the experiments at these rather high B values. However, one cannot discard this might be due to the strong Landau damping existing in this energy region. We remark that within LSDA, a spin wave instability of similar type has been found in parabolically confined dots at $B = 0$ [31] whose origin is a spontaneous breaking of the circular symmetry, which could also be the origin of the spin dipole instability.

It is of some interest to look at the effect that a moderate temperature may have on the spin response of the $N = 210$ dot at high magnetic fields. As an example, we have considered $B = 5.14$ T ($\nu = 2$), and $B = 7$ T, and have obtained the spin and charge responses at $T = 2$ K.

The results are displayed in Fig. 10. Apart from the well known T-independence of the charge response at low temperatures, the figure shows that the low energy spin peak recovers at $T = 2$ K, no longer lying at zero energy [38]. We do not display the results for $\nu = 2$, since the response remais essentially unchanged. The different behavior is due to the quite distinct sp spectrum of these configurations, as can be seen from Figs. 2(b) and 5(d) of Ref. [20], which makes the $B = 7$ T state very sensitive to thermal effects (see Fig. 7 of Ref. [20]). It is a general rule that integer ν states are rather inert thermally because for them the chemical potential lies between Landau bands and consequently, very few sp states are affected by a change in T if it is much smaller than the sp energy gap, which is the present case. The situation is completely different for B values that do not correspond to integer ν values. In this situation, the chemical potential is on top of a Landau band, and the changes are sizeable because of the large density of states around the chemical potential (see Figs. 2, 5 and 7 of Ref. [20]).

Finally, we would like to comment on the density dipole mode. For the parabolic confining potential, Figs. 6 and 9 show the well-known result that the density response yields the classical law represented by the first of Eqs. 24. For the disk confining potential, that law is also fairly obeyed, particularly by the negative B dispersion branch. We have systematically found that the positive B dispersion branch is fragmented, especially for the $N = 210$ dot. Comparing our calculations for the $N = 25$ dot with the experimental results [2], we conclude that a parabolic potential is better suited than a disk potential to represent the physical situation. On the contrary, the confining potential of the $N = 210$ dot is not parabolic, and this is the origin of the second upper branch found in the experiment. The non-harmonicity of the confining potential has been presented in Ref. [5] as the origin of that branch on the basis of a Hartree+random phase approximation (RPA) calculation in dots with $N \leq 30$ (see also Ref. [39]). Our TDLSDA calculation supports that interpretation.

It is worth to see that the density response of such a large dot displayes two instead of one satellite branch (compare with the results for the small $N = 25$ dot), i.e., TDLSDA seems to produce a high frequency peak that is more fragmented than in the experiments. We want to remark that the upper branches disappear at intense B, in agreement with the experimental findings. This gives further support to our explanation and that of [5] about the origin of these branches. Indeed, at higher magnetic fields one expects that the harmonic

cyclotron potential dominates over the other contributions in the KS equation.

IV. THE VIBRATING POTENTIAL MODEL

The intuitive idea behind TDLSDA is that a small amplitude time-dependent variation of the mean field around the static equilibrium configuration produces an oscillation in the electronic density, which causes a small amplitude collective motion of the system. This motion is self sustained if the induced density is precisely that needed to generate the oscillating potential. The vibrating potential model naturally arises when one considers the first iteration of the (perturbed mean field) $\longleftrightarrow$ (induced density) selfconsistent scheme. In homogenous systems, where translational invariance determines the shape of the induced density, it yields the exact solution. In finite systems (nuclei, metal clusters, dots), the model provides a useful approximation.

Using the general method described in the Appendix, we consider the following VPM Hamiltonian

$$
\begin{aligned}
H &= \sum_{i=1}^{N} \left[-\frac{\nabla^2}{2} + \frac{1}{2}\omega_c \ell_z + \frac{1}{8}\omega_c^2 r^2 + \frac{1}{2}\omega_{xc}^2 r^2 + (\frac{1}{2}g^*\mu_B B + \frac{V_\sigma}{\rho_0}m_0)\sigma_z + \delta V(\vec{r},t) \right]_i \\
&= \sum_{i=1}^{N} \left[H_0(\vec{r_i}) + \delta V(\vec{r_i},t) \right]
\end{aligned}
\tag{17}
$$

with a time-oscillating potential

$$
\delta V(\vec{r},t) = -\frac{1}{N} \left[(\omega_{xc}^2 + \omega_0^2)\langle \sum_k x_k \rangle x - 2\frac{V_\sigma}{\langle r^2 \rangle}\langle \sum_k x_k \sigma_z^k \rangle x\sigma_z \right] \quad ,
\tag{18}
$$

where $\omega_0^2 = \pi\, n_s/R$. The time dependence of $\delta V(\vec{r},t)$ is in the $\langle ... \rangle$ spatial foldings with the densies induced by a time dependent external field, see the Appendix. To get the static part H_0 of the Hamiltonian Eq. (17) we have assumed an exact cancellation between the Hartree and the external potentials, and have taken a parabolic approximation for the exchange-correlation potential at $B = 0$. V_σ is the exchange-correlation constant introduced in the Appendix.

This VPM Hamiltonian can now be solved analytically within RPA by finding the operators O^+ solution of the equations of motion

$$
[H, O^+] = \omega\, O^+ \quad .
\tag{19}
$$

We have used the methods illustrated in Ref. [35] to compute the conmutators with the Hamiltonian as

$$
[H, O] = [H_0, O] + \delta V(O) \quad ,
\tag{20}
$$

where H_0 is the static Hamiltonian and $\delta V(O)$ the variation arising from the induced densitites. It can be shown that the solutions to Eq. (19) are given by

$$O_\pm^{\rho+} = \frac{1}{2}\sqrt{\frac{\bar{\omega}}{N}}\left(Q_\pm - \frac{i}{\bar{\omega}}P_\pm\right)$$

$$O_\pm^{m+} = \frac{1}{2}\sqrt{\frac{\bar{\omega}_\sigma}{N[1-(\frac{2S_z}{N})^2]}}\left[(Q_\pm^\sigma - \frac{i}{\bar{\omega}_\sigma}P_\pm^\sigma) - \frac{2S_z}{N}(Q_\pm - \frac{i}{\bar{\omega}_\sigma}P_\pm)\right] \ , \tag{21}$$

where

$$Q_\pm = \sum_{i=1}^{N}(x_i \pm y_i) \ , \ P_\pm = \sum_{i=1}^{N}(p_{xi} \pm p_{yi})$$

$$Q_\pm^\sigma = \sum_{i=1}^{N}(x_i \pm y_i)\sigma_z^i \ , \ P_\pm^\sigma = \sum_{i=1}^{N}(p_{xi} \pm p_{yi})\sigma_z^i \tag{22}$$

and

$$\bar{\omega} = \sqrt{\omega_o^2 + \frac{\omega_c^2}{4}} \ , \ \bar{\omega}_\sigma = \sqrt{\omega_{xc}^2 + \frac{2V_\sigma}{\langle r^2 \rangle} + \frac{\omega_c^2}{4}} \ . \tag{23}$$

The corresponding frequencies are

$$\omega_\pm^\rho = \bar{\omega} \pm \frac{\omega_c}{2}$$

$$\omega_\pm^m = \bar{\omega}_\sigma \pm \frac{\omega_c}{2} \ , \tag{24}$$

and it is easy to verify that

$$[L_z, O_\pm^+] = \pm O_\pm^+ \ . \tag{25}$$

The states $|\omega_\pm^{\rho,m}\rangle \equiv O_\pm^{\rho,m\,+}|0\rangle$ are orthonormal and carry an orbital angular momentum $L_0 \pm 1$ and a spin S_0, where L_0 and S_0 are the orbital and spin angular momenta of the ground state, respectively.

The charge dipole and spin dipole strengths are distributed among the above states as follows:

$$|\langle 0|D_\rho|\omega_+^\rho\rangle|^2 = |\langle 0|D_\rho|\omega_-^\rho\rangle|^2 = \frac{1}{4}\frac{N}{\bar{\omega}}$$

$$|\langle 0|D_m|\omega_+^\rho\rangle|^2 = |\langle 0|D_m|\omega_-^\rho\rangle|^2 = \frac{S_z^2}{\bar{\omega}N}$$

$$|\langle 0|D_\rho|\omega_+^m\rangle|^2 = |\langle 0|D_\rho|\omega_-^m\rangle|^2 = 0$$

$$|\langle 0|D_m|\omega_+^m\rangle|^2 = |\langle 0|D_m|\omega_-^m\rangle|^2 = \frac{1}{4}\frac{N}{\bar{\omega}_\sigma}\left[1-\left(\frac{2S_z}{N}\right)^2\right] \ . \tag{26}$$

It is a simple matter to check that the above solutions exhaust the sum rules Eq. (15).

This vibrating potential model reproduces the most salient features of the full self-consistent calculation. Its parabolic form guarantees that Kohn's theorem is fulfilled, and as a consequence the response to D_ρ is shared by just two peaks, which have the same strenght $N/4\bar{\omega}$. Accordingly, the spin dipole modes $|\omega_\pm^m\rangle$ are not excited by the dipole operator D_ρ and the corresponding matrix elements vanish. Another consequence of Kohn's theorem is

that within VPM only the density mode contributes to the mixed response. The model predicts that in the mixed channel, the charge dipole modes $|\omega_\pm^\rho\rangle$ are excited with the same strength $S_z^2/\bar{\omega}N$ by the spin dipole operator D_m.

In the spin channel, the spin dipole operator D_m excite both the charge $|\omega_\pm^\rho\rangle$ and spin $|\omega_\pm^m\rangle$ modes. These peaks have strengths $|\langle 0|D_m|\omega_\pm^m\rangle|^2 = N[1 - (2S_z/N)^2]/4\bar{\omega}_\sigma$ and $|\langle 0|D_m|\omega_\pm^m\rangle|^2 = S_z^2/\bar{\omega}N$, which are the same for both $\Delta L_z = \pm1$ branches. Finally, when $S_z = 0$ the density and spin modes are decoupled, and when the system is fully polarized, i.e., $2S_z = N$, all the strength is transferred to the density channel.

Besides this qualitative agreement, the strengths given by the first and last lines of Eq. (26) agree well with the result of the microscopic calculation. Also the ratio $(2S_z/N)^2$ between the strength of the density peaks excited by D_m and D_ρ is reproduced.

The second Eq. (24) can be used to determine the energy of the spin dipole mode $\omega_\pm^m$ as the first one has been often used in the case of the charge dipole mode, if we fix the parameters entering that equation. We first take $\omega_{xc}^2 = 2\pi n_s/R^2$. This estimate is obtained identifying the kinetic energy per particle in the exchange-correlation harmonic oscillator potential with that of the 2D Fermi gas $\pi n_s/2$, and approximating the mean square radius of the dot $\langle r^2 \rangle$ by $R^2/2 = r_s^2 N/2$, where $r_s^2 = 1/(\pi n_s)$. One then gets

$$\omega_\pm^m = \sqrt{\frac{4}{r_s^4 N}\left(r_s^2 V_\sigma + \frac{1}{2}\right) + \frac{\omega_c^2}{4}} \pm \frac{\omega_c}{2} \quad . \tag{27}$$

The value of $r_s^2 V_\sigma$ is related to the spin susceptibility of the two-dimensional electron gas $\chi_0/\chi = r_s^2 V_\sigma + 1$ (Ref. [30]). Eq. (27) yields values for $\omega_\pm^m$ which agree with the TDLSDA ones. In particular, we have checked that the N dependence of the $B = 0$ calculations reported in Ref. [14] is well reproduced taking $r_s^2 V_\sigma \simeq -0.3$ at $r_s \simeq 1$.

Notice that the N and r_s dependence of energy of the spin mode is different from that of the density mode, which is given by

$$\omega_\pm^\rho = \sqrt{\frac{1}{r_s^3 N^{1/2}} + \frac{\omega_c^2}{4}} \pm \frac{\omega_c}{2} \tag{28}$$

when we take for the frequency ω_0 of the confining potential the estimate $\omega_0 = r_s^{-3/2} N^{-1/4}$ obtained from the disk potential in the $r/R << 1$ limit.

We report in Fig. 11 the energies given by Eq. (27) for a dot of $N = 200$ together with the experimental spin dipole mode of Ref. [13]. A value of $r_s = 0.65$ has been used which yields $r_s^2 V_\sigma = -0.24$. From the figure one sees that the negative B dispersion experimental branch is fairly well reproduced. It has been experimentaly found that the dipole positive B dispersion branch, as well as other spin modes laying at higher energies are rather B independent, so one should not expect that the simple model leading to Eq. (27) reproduces this behavior.

The VPM works better for large than for small dots. This is not surprising in view of the approximations leading to VPM. It can be seen comparing with the TDLSDA results in Figs. 6-9 after being advised that for the $N = 5$ dot, the dotted line representing $|\omega_+^m\rangle$ is not passing near the main peak energies of the positive B dispersion branch, but near to the satellite peak energies.

Eq. (27) shows that within VPM, the spin instability at $B = 0$ mentioned in the previous Section occurs when $r_s^2 V_\sigma + 1/2 \leq 0$. In the 2D electron gas it happens when $r_s^2 V_\sigma + 1 \leq 0$ [30]. The difference between these conditions is due to finite size effects, which are crucial to determine the values of r_s at which the instability appears in quantum dots. Whereas in the bulk the spin instability sets in at $r_s \simeq 37$, Eq. (27) yields that in dots it happens at $r_s \sim 3$. This value is well within the range of those found in Ref. [31] for the onset of a spin density wave instability in small magic dots, and a factor of two larger than the typical values obtained in that reference for open shell dots.

V. SUMMARY AND OUTLOOK

In this work we have thoroughly discussed the longitudinal dipole response of quantum dots. We have shown that TDLSDA is able to reproduce the main features of the experimental results. In particular, we have found that the density and spin modes are clearly coupled in the spin channel if the system is partially polarized, and that the spin modes are especially soft for even filling factors. We have determined that at $B \neq 0$ the spin response lies in the energy region of single electron excitations.

We predict that the frequency of the spin edge magnetoplasmon presents an oscillatory behavior as a function of ν, and that the spin dipole mode becomes unstable at B values in the $1 < \nu \leq 2$ region.

Our numerical scheme has allowed us to study large size dots whose spectrum has been experimentally studied in detail, instead of relying on extrapolation of the results obtained for small size dots. This is crucial to identify the ν behavior of physical quantities like excitation frequencies.

TDLSDA can be easily applied to other multipole spin and density excitations. This is relevant in view that recent experiments have been able to identify the spin monopole and quadrupole modes, and likely charge modes different form dipole [13]. Work to extend the present study to other multipolarities, and to study the spin transverse channel in large dots along the line described in Ref. [40] is under way.

APPENDIX:

In this Appendix we provide a general derivation of the VPM Hamiltonian leading to Eq. (17). We start from the density and magnetization variations

$$(\delta\rho(\vec{r}, t), \delta m(\vec{r}, t)) = \alpha(t)(\nabla_x \rho_0(\vec{r}), \nabla_x m_0(\vec{r}))$$
$$+ \beta(t)(\nabla_x m_0(\vec{r}), \nabla_x \rho_0(\vec{r})) \quad , \tag{A1}$$

obtained from the static ground state density ρ_0 and spin magnetization m_0 through the unitary transformations $e^{\alpha(t)\sum_i \nabla_x^i}$ and $e^{\beta(t)\sum_i \nabla_x^i \sigma_z^i}$, respectively, in the limit of small deformations. In Eq. (A1) $\alpha(t)$ and $\beta(t)$ give the amplitude of the oscillations.

Assuming for the exchange-correlation potentials the forms $V^{xc} = V^{xc}(\rho)$ and $W^{xc} = mV_\sigma/\rho_0$, which amounts to expand $\mathcal{E}(\rho, m)$ around $m = 0$ up to m^2 order and identifying V_σ with $\rho_0 \partial^2 \mathcal{E}_{xc}(\rho_0, m)/\partial m^2|_{m=0}$, one obtains the variation in the one body potential of Eq. (1) induced by the density variations (A1):

$$\delta V(\vec{r}_i, t) = \alpha(t)\left(Q(\vec{r}_i) + \frac{2S_z}{N}Q_\sigma(\vec{r}_i)\,\sigma_z^i\right) + \beta(t)\left(\frac{2S_z}{N}Q(\vec{r}_i) + Q_\sigma(\vec{r}_i)\,\sigma_z^i\right) \quad , \qquad (A2)$$

where

$$Q(\vec{r}) = \left(\nabla_x V_0 + \int \frac{\nabla_x \rho_0(\vec{r})}{|\vec{r} - \vec{r}'|}d\vec{r}'\right)_i \quad , \qquad Q_\sigma(\vec{r}) = \frac{V_\sigma}{\rho_0}\nabla_x \rho_0(\vec{r}) \qquad (A3)$$

with $V_0 = V^{xc}(\rho = \rho_0)$. We have further assumed that $m_0 = \frac{2S_z}{N}\rho_0$ in the gs. Using the results

$$\left\langle\sum_i Q(\vec{r}_i)\right\rangle \equiv \int Q(\vec{r})\delta\rho(\vec{r}, t)d\vec{r} = -\left(\alpha(t) + \frac{2S_z}{N}\beta(t)\right)\int \rho_0(\vec{r})\nabla_x Q(\vec{r})d\vec{r}$$

$$\left\langle\sum_i Q_\sigma(\vec{r}_i)\sigma_z^i\right\rangle \equiv \int Q_\sigma(\vec{r})\delta m(\vec{r}, t)d\vec{r} = -\left(\frac{2S_z}{N}\alpha(t) + \beta(t)\right)\int \rho_0(\vec{r})\nabla_x Q_\sigma(\vec{r})d\vec{r} \qquad (A4)$$

it is then possible to write the variations in the one-body potential of Eq. (1) in the separable form

$$\delta V(\vec{r}_i, t) = \frac{-1}{1 - (\frac{2S_z}{N})^2}\left[\left(\frac{\langle\sum_i Q(\vec{r}_i)\rangle}{\int(\nabla_x Q(\vec{r}))\rho_0(\vec{r})d\vec{r}} - \frac{2S_z}{N}\frac{\langle\sum_i Q_\sigma(\vec{r}_i)\sigma_z^i\rangle}{\int(\nabla_x Q_\sigma(\vec{r}))\rho_0(\vec{r})d\vec{r}}\right)\times\right.$$

$$\left(Q(\vec{r}_i) + \frac{2S_z}{N}Q_\sigma(\vec{r}_i)\sigma_z^i\right) + \left(\frac{\langle\sum_i Q_\sigma(\vec{r}_i)\sigma_z^i\rangle}{\int(\nabla_x Q_\sigma(\vec{r}))\rho_0(\vec{r})d\vec{r}} - \frac{2S_z}{N}\frac{\langle\sum_i Q(\vec{r}_i)\rangle}{\int(\nabla_x Q(\vec{r}))\rho_0(\vec{r})d\vec{r}}\right)\times$$

$$\left.\left(\frac{2S_z}{N}Q(\vec{r}_i) + Q_\sigma(\vec{r}_i)\sigma_z^i\right)\right] \quad . \qquad (A5)$$

Following Ref. [29], one could now express the various responses to an oscillating potential of the form (A5) in terms of the independent particle response functions $\chi^0_{\sigma,\sigma'}$ through RPA-type equations. However, our aim here is to develop an analytical model which allows us to understand in a simple way the numerical results of Section III. To this end we take in (A3) a harmonic oscillator for the one-particle potential $V_0 = \frac{1}{2}\omega_{xc}^2 r^2$ to simulate the short range effects, and a step function for the electronic density ρ_0 entering the long range contribution $\int \nabla_x \rho_0(\vec{r})/|\vec{r} - \vec{r}'|d\vec{r}'$. We also assume $Q_\sigma(\vec{r}_i) \simeq -2V_\sigma x_i/\langle r^2\rangle$. Equation (17) of Section IV is then obtained.

REFERENCES

[1] Ch. Sikorski and U. Merkt, Phys. Rev. Lett. **62**, 2164 (1989).

[2] T. Demel, D. Heitmann, P. Grambow, and K. Ploog, Phys. Rev. Lett. **64**, 788 (1990).

[3] D.A. Broido, K. Kempa, and B. Bakshi, Phys. Rev. B **42**, 11400 (1990).

[4] P. A. Maksym and T. Chakraborty, Phys. Rev. Lett. **65**, 108 (1990); Phys. Rev. B **45**, 1947 (1992).

[5] V. Gudmundsson and R.R. Gerhardts, Phys. Rev. B **43**, 12098 (1991).

[6] V. Shikin, S. Nazin, D. Heitmann, and T. Demel, Phys. Rev. B **43**, 11 903 (1991).

[7] V. Gudmundsson and J.J. Palacios, Phys. Rev. B **52**, 11266 (1995).

[8] L. Jacak, P. Hawrylak, and A. Wójs, *Quantum dots* (Springer, Berlin, 1998).

[9] W. Kohn, Phys. Rev. **123**, 1242 (1961).

[10] R. Strenz, U. Bockelmann, F. Hirler, G. Abstreiter, G. Böhm, and G. Weimann, Phys. Rev. Lett. **73**, 3022 (1994).

[11] C. Schüller, G. Biese, K. Keller, C. Steinebach, D. Heitmann, P. Grambow, and K. Eberl, Phys. Rev. B **54**, R17304 (1996).

[12] D.J. Lockwood, P. Hawrylak, P.D. Wang, C.M. Sotomayor Torres, A. Pinczuk, and B. S. Dennis, Phys. Rev. Lett. **77**, 354 (1996).

[13] C. Schüler, K. Keller, G. Biese, E. Ulrichs, L. Rolf, C. Steinebach, and D. Heitmann, Phys. Rev. Lett. **80**, 2673 (1998).

[14] Ll. Serra and E. Lipparini, Europhys. Lett. **40**, 667 (1997).

[15] A. Richter, *Nuclear Physics with electromagnetic interactions.* H. Arenhovel and D. Drechsel, editors, Lect. Notes Phys. **108**, 19 (1979).

[16] Ll. Serra, R.A. Broglia, M. Barranco and J. Navarro, Phys. Rev. A **47**, 1601 (1993).

[17] E. Lipparini and M. Califano, Z. Phys. D **37**, 365 (1996).

[18] Ll. Serra, M. Pi, A. Emperador, M. Barranco, and E. Lipparini. Contribution to the 9th International Symposium on Small Particles and Inorganic Clusters, Lausanne (Switzerland), 1-5 September 1998.

[19] E. Lipparini, N. Barberan, M. Barranco, M. Pi and Ll. Serra, Phys. Rev. B **56**, 12375 (1997).

[20] M. Pi, M. Barranco, A. Emperador, E. Lipparini, and Ll. Serra, Phys. Rev. B **57**, 14783 (1998).

[21] M. Ferconi and G. Vignale, Phys. Rev. B **50**, 14722 (1994).

[22] M. Ferconi, M. R. Geller, and G. Vignale, Phys. Rev. B **52**, 16357 (1995).

[23] O. Heinonen, M. I. Lubin, and M. D. Johnson, Phys. Rev. Lett **75**, 4110 (1995).

[24] M. Ferconi and G. Vignale, Phys. Rev. B **56**, 12108 (1997).

[25] M. I. Lubin, O. Heinonen, and M. D. Johnson, Phys. Rev. B **56**, 10373 (1997).

[26] J. J. Palacios, L. Martín-Moreno, G. Chiappe, E. Louis, and C. Tejedor, Phys. Rev. B **50**, 5760 (1994).

[27] D. Levesque, J. J. Weiss, and A. H. MacDonald, Phys. Rev. B **30**, 1056 (1984).

[28] A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975).

[29] E. Lipparini and S. Stringari, Z. Phys. D **18**, 193 (1991).

[30] B. Tanatar and D.M. Ceperley, Phys. Rev. B **39**, 5005 (1989)

[31] M. Koskinen, M. Manninen and S.M. Reimann, Phys. Rev. Lett. **79**, 1389 (1997).

[32] U. von Barth and L. Hedin, J. Phys. C **5**, 1629 (1972).

[33] A.R. Williams and U. von Barth, *Theory of the Inhomogenous Electron Gas*, pag. 231. S. Lundqvist and N.H. March , editors. (Plenum, New York, 1983).

[34] A. K. Rajagopal, Phys. Rev. B **17**, 2980 (1978).

[35] E. Lipparini and S. Stringari, Phys. Rep. **175**, 103 (1989).

[36] C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).

[37] K. Bollweg, T. Kurth, D. Heitmann, V. Gudmundsson, E. Vasiliadou, P. Grambow, and K. Ebert, Phys. Rev. Lett. **76**, 2774 (1996).

[38] The results at $T = 2$ K displayed in Fig. 10 have been obtained with an imaginary part of the frequency $\eta = 0.015$ instead of the 0.006 value used everywere else, to make sure that the condition $\eta > \Delta\omega > k_B T$ is fulfilled.

[39] Z. L. Ye and E. Zaremba, Phys. Rev. B **50**, 17217 (1994).

[40] E. Lipparini and Ll. Serra, Phys. Rev. B **57**, R6830 (1998).

FIGURES

FIG. 1. Dipole strength function (effective atomic units) of the $N = 5$ dot as a function of frequency (meV). Solid and dotted lines correspond to the density response to D_ρ, and to the spin response to D_m, respectively. Dashed lines represent the free particle strength function.

FIG. 2. Same as Fig. 1 for $N = 25$.

FIG. 3. Same as Fig. 1 for $N = 210$.

FIG. 4. Same as Fig. 2 using a parabolic potential with $\omega_0 = 2.78$ meV instead of a disk confining potential.

FIG. 5. Mixed $S_{m\rho}(\omega)$ response function (effective atomic units) of the $N = 210$ dot at $\nu = 3$. The arrows indicate the density and spin mode peaks.

FIG. 6. B dispersion of the main peaks of the $N = 5$ spectrum. The circles correspond to density modes and the triangles to spin modes. The solid symbols correspond to $\nu = 1$. The lines represent the VPM B dispersion laws with fitted value at $B = 0$.

FIG. 7. B dispersion of the main peaks of the $N = 25$ spectrum for a disk confining potential. The circles correspond to density modes and the triangles to spin modes. The crosses are experimental points from Ref.[2]. The lines represent the VPM B dispersion laws with fitted values at $B = 0$. The insert shows the negative B dispersion branch of the spin mode. From left to right the solid symbols correspond to $\nu = 6$ to 1.

FIG. 8. B dispersion of the main peaks of the $N = 210$ spectrum for a disk confining potential. The circles correspond to density modes and the triangles to spin modes. The crosses are experimental points from Ref.[2]. The lines represent the VPM B dispersion laws with fitted values at $B = 0$. The insert shows the negative B dispersion branch of the spin mode. From left to right the solid symbols correspond to $\nu = 8$ to 1.

FIG. 9. Same as Fig. 7 using a parabolic confining potential with $\omega_0 = 2.78$ meV instead of a disk confining potential.

FIG. 10. Dipole responses for the $N = 210$ dot at $B = 7$ T. Top panel, $T = 0.1$ K. Bottom panel, $T = 2$ K. Lines as indicated in Fig. 1.

FIG. 11. B dispersion of the spin dipole mode within VPM. Experimental points are from Ref. [13].

(a) B = 0 T
(b) B = 1 T
(c) B = 2 T
(d) B = 3 T
(e) B = 3.5 T
(f) B = 4 T, ν = 1

ω (meV)

Figure showing spectral functions ω (meV) for six magnetic field values:
(a) B = 0 T
(b) B = 0.59 T, $\nu = 6$
(c) B = 1.19 T, $\nu = 3$
(d) B = 1.78 T, $\nu = 2$
(e) B = 2.5 T
(f) B = 3.56 T, $\nu = 1$